ITERATIVE METHOD FOR SUPERSONIC FLOW LAMINAR BOUNDARY LAYER INTERACTION IN THE PRESENCE OF SEPARATION

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The classical two-dimensional compressible boundary-layer equations supplemented by a relation describing the interaction of boundary layer with external inviscid flow (see, e.g., [1]) are treated as the governing equations in one of the methods to study the viscous-inviscid interaction. It is then necessary in the case of supersonic flow to specify certain downstream boundary conditions for the closure of the governing system, i.e., it is a boundary-value problem (e.g., [2]). The "shooting" technique for parameters at the beginning of the computational region to obtain the solution satisfying such a condition usually requires large computer time since the integral curves are highly sensitive to small changes in upstream boundary conditions. A more effective method is the algorithm of global relaxations of pressure distribution along the entire computational region [1]. A numerical method to compute supersonic interacting boundary layer in the presence of separation is presented in this paper.

1. It is convenient to write the governing equations for a compressible, two-dimensional boundary layer of a perfect gas by introducing the following nondimensional variables:

$$X=\frac{s}{L},\quad Y=\frac{u_e}{\sqrt{2X\,\mathrm{Re}_\infty\mu_\infty}}\int_0^n\rho\,dn,\quad F=\frac{u}{u_e},$$
$$G=\frac{H}{H_e},\quad V=\frac{\rho v\sqrt{2X\,\mathrm{Re}_\infty}}{\rho_\infty u_\infty}+F\frac{\partial Y}{\partial X}.$$

Here s, n represent orthogonal coordinate system associated with the surface of the body; u, v are velocity components; μ is the coefficient of viscosity; ρ is the density; H is the total enthalpy; Re = $u\rho L/\mu$ is the Reynolds number; L is the characteristic body length. The indices e and ∞ refer to conditions outside the boundary layer and in the supersonic free stream. Assuming a linear relation between viscosity and temperature $\mu/\mu_\infty = CT/T_\infty$ and constant Prandtl number, the basic equations and the boundary conditions can be written in the form

$$\begin{gathered}V'+F+2X\dot{F}=0,\\ \frac{\rho_e u_e\mu_e}{\rho_\infty u_\infty\mu_\infty}F''-VF'+2\frac{d\ln M_e}{d\ln X}(G-F^2)=2XF\dot{F},\\ \frac{\rho_e u_e\mu_e}{\rho_\infty u_\infty\mu_\infty}G''-VG'+\frac{\rho_e u_e^3\mu_e}{2\rho_\infty u_\infty\mu_\infty H_e}\left(1-\frac{1}{\Pr}\right)(F^2)''=2XF\dot{G},\\ V(X,0)=F(X,0)=0,\ G(X,0)=G_w(X)\ \text{for}\ G'(X,0)=0,\ F(X,\infty)=\\ =G(X,\infty)=1,\end{gathered}\qquad(1.1)$$

where M is the Mach number; the index w denotes the surface of the body; dots and primes denote differentiation with respect to X and Y.

In order to take into account viscous interaction, an effective body thickness $\delta = y_w + \delta^*$ is introduced which is the sum of the surface ordinary $y_w(X)$ and boundary-layer displacement thickness $\delta^*(X)$ nondimensionalized using the length scale L. Then the distribution of flow variables at the outer edge of the boundary layer, determining the coefficients of the system (1.1), is dependent on the inclination of the streamlines of the outer isentopic flow past the effective body. In particular, the variation of Mach number $M_e(X)$ or pressure can

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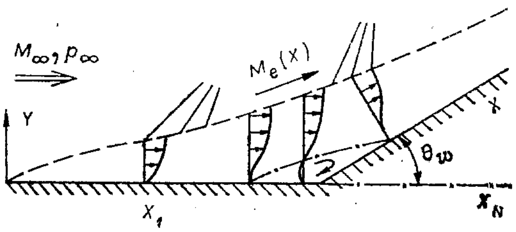

Fig. 1

be computed using $\dot{\delta}(X)$ from the interaction equation representing the Ackeret, Prandtl—Meyer equation or the tangential wedge approximation. The presence of the term $\ddot{\delta}(X)$ in the expression for the pressure gradient makes it necessary to solve the boundary-value problem (1.1) with both upstream and downstream boundary conditions. The solution of the problem with weak interaction at the initial segment of the body is carried out in this paper to obtain velocity and enthalpy profiles and also δ_1 and p_1 on the first computational characteristic $X = X_1$. When $X = X_N$ in the downstream region of the flow it is generally required to make the pressure equal to a previously specified value: $p(X_N) = p_N$.

2. In order to solve the boundary-value problem (1.1), an iterative procedure with successive approximations for the distribution of $\delta(X)$ is used. For each iteration it is required that the function $\delta(X)$ satisfies the previously specified boundary conditions

$$\delta(X_1) = \delta_1, \quad \dot{\delta}(X_N) = \dot{\delta}_N, \tag{2.1}$$

where, according to the interaction equation, $\dot{\delta}_N$ corresponds to the given value of the base pressure at $X = X_N$. Let the distribution $\delta^{(n)}(X)$ satisfying condition (2.1) be specified at the beginning of the n-th iteration. Coefficients of the system (1.1) are determined using $\dot{\delta}^{(n)}(X)$ and $\ddot{\delta}^{(n)}(X)$ after which the initial equations are numerically integrated. In the case of attached flow the traditional marching technique is used and to compute separated flows relaxation technique is used with variable direction scheme in accordance with change in the direction of propagation of disturbances in the reverse flow regions [3]. The computed flow fields $F^{(n)}(X, Y)$ and $G^{(n)}(X, Y)$ make it possible to obtain the distribution of displacement thickness and effective body thickness $\delta_p^{(n)}(X)$. In order to determine the next approximation $\delta^{(n+1)}(X)$ the following procedure is used: $\delta^{(n+1)}(X) = \delta^{(n)}(X) + \Delta^{(n)}(X)$, where the function $\Delta^{(n)}(X)$ is found from the solution of the boundary-value problem

$$\ddot{\Delta}^{(n)} - \alpha_1\Delta^{(n)} = \alpha_2\left(\delta^{(n)} - \delta_p^{(n)}\right), \quad \Delta^{(n)}(X_1) = 0, \quad \dot{\Delta}^{(n)}(X_N) = 0. \tag{2.2}$$

Here α_1 and α_2 are positive constants. The iteration is continued until $\delta^{(n)}$ and $\delta_p^{(n)}$ satisfy the specified relative accuracy ε in the entire computational interval $[X_1, X_N]$.

The function $\delta^{(n+1)}(X)$ satisfies conditions (2.1) and meets the natural requirement $\delta^{(n+1)}(X) \to \delta^{(n)}(X)$ if $\delta^{(n)}(X) - \delta_p^{(n)}(X) \to 0$ on $[X_1, X_N]$. Observe that the equation of the same functional form as (2.2) can be obtained using integral form of initial boundary-layer equation. One of the merits of this method is that Eq. (2.2) has a very simple form and in order to solve it information is required on the computational flow field only in the form of the integral function $\delta_p^{(n)}(X)$. The initial approximation $\delta^{(1)}(X)$ necessary for beginning the procedure can, in the general case, be any smooth distribution satisfying boundary conditions (2.2).

3. The practically important case of the flow past a compression corner was considered in all the computational examples. The body configuration and the flow pattern are shown in Fig. 1. The equation of pressure to a value corresponding to inviscid flow past semi-infinite wedge is used as the downstream boundary condition. Transformation of the transverse coordinate $Y_1 = Y/(1 + Y)$ was made in order to improve the accuracy of computations in the near-wall flow region. Computations were carried out using finite-difference method with second-order accuracy in ΔX and ΔY_1 and step sizes $\Delta X = \Delta Y_1 = 0.02$. Numerical analysis of the effect of parameter α_1 and α_2 on convergence rate showed that for attached flows their values should be of the order of 100 and of the order of 0.1 for the separated region.

The first series of computations corresponded to the case considered in [2] ($M_\infty = 3.0$, $Re_\infty = 10^5$, $G_w = 0.5$, $Pr = 0.72$). The vertex of the corner with inclination θ_w was located at the point $X = 1.55$. Ackeret's formula was used as the condition for interaction. Figure 2 shows pressure distribution along the surface at inclinations 9, 10, and 11°. The distribution of the parameter c_f^* is shown in Fig. 3, where $c_f^* = c_f\sqrt{Re_\infty}/2$ (c_f is the skin friction

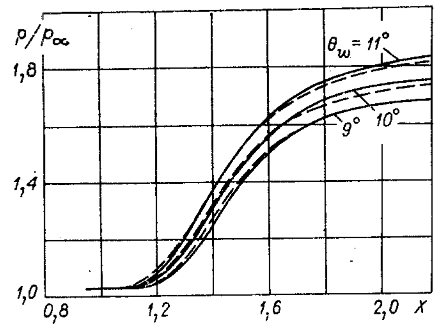

Fig. 2

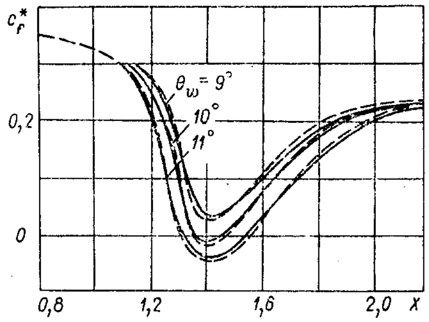

Fig 3

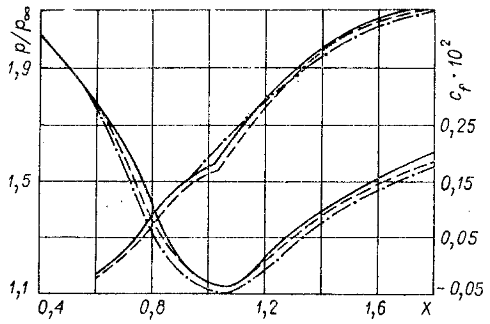

Fig. 4

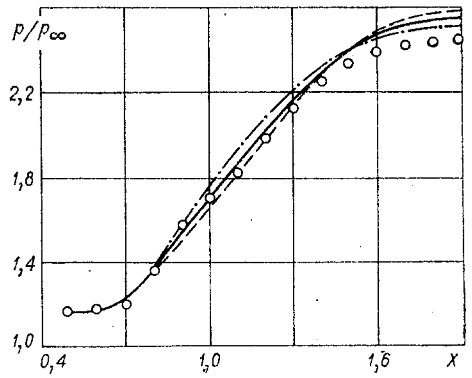

Fig. 5

drag coefficient based on free stream parameters). The results obtained (continuous curves) agree well with computed data [2] (dashed lines).

The following two numerical cases use tangential wedge approximation for pressure. The initial data for the first case are: $M_\infty = 3.0$, $Re_\infty = 1.68\cdot10^4$, $G_w = 1$, $Pr = 1$, $\theta_w = 10°$. The continuous lines in Fig. 4 show the pressure distribution and skin friction coefficient along the body along with solutions to Navier–Stokes equations [4] (dashed lines) and [5] (dashed–dotted lines). A comparison shows a good agreement of all the data. The second case corresponds to the conditions in an experimental study [6]: $M_\infty = 4.0$, $Re_\infty = 6.8\cdot10^4$, $G'_w = 0$ (adiabatic wall), $Pr = 0.72$, $\theta_w = 10°$. The surface pressure distribution obtained in the present work is shown in Fig. 5 along with numerical solutions to Navier–Stokes equation [5] (dashed line), numerical results of [1] (dashed–dotted line), and experimental data [6] (circles). A good agreement of computed and experimental results is observed.

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